SUBJECT CODE	SUBJECT
A-15-18	**MATHEMATICAL SCIENCES**

HALL TICKET NUMBER							

PAPER
II

OMR SHEET NUMBER					

NUMBER OF QUESTIONS
100

DURATION	MAXIMUM MARKS	NUMBER OF PAGES
2 HOURS	**200**	**24**

This is to certify that, the entries made in the above portion are correctly written and verified.

Candidates Signature

Name and Signature of Invigilator

Instructions for the Candidates

1. Write your Hall Ticket Number in the space provided on the top of this page.
2. This paper consists of hundred multiple-choice type of questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested **<u>to open the booklet and compulsorily examine it as below</u>** :
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal and do not accept an open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.**
   (iii) After this verification is over, the Test Booklet Number should be entered in the OMR Sheet and the OMR Sheet Number should be entered on this Test Booklet.
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : (A) (B) ● (D)
   where (C) is the correct response.
5. Your responses to the items are to be indicated in the **OMR Answer Sheet given to you**. If you mark at any place other than in the circle or half circle or semi circle in the Answer Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. **The candidate must handover the OMR Answer Sheet to the invigilators at the end of the examination compulsorily** and must not carry it with you outside the Examination Hall. The candidate is allowed to take away the carbon copy of OMR Sheet and used Question paper booklet at the end of the examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table etc., is prohibited.**
12. **There is no negative marks for incorrect answers.**

అభ్యర్థికి సూచనలు

1. ఈ పుట పై భాగంలో ఇవ్వబడిన స్థలంలో మీ హాల్‌టికెట్ నంబరు రాయండి.
2. ఈ ప్రశ్న పత్రము వంద (100) బహుళైచ్ఛిక ప్రశ్నలను కలిగి ఉంది.
3. పరీక్ష ప్రారంభమున ఈ ప్రశ్నాపత్రము మీకు ఇవ్వబడుతుంది మొదటి ఐదు నిమిషములలో **<u>ఈ ప్రశ్నాపత్రమును తెరిచి కింద తెలిపిన అంశాలను తప్పనిసరిగా సరిచూసుకోండి.</u>**
   (i) ఈ ప్రశ్న పత్రమును చూడడానికి కవర్ పేజి అంచున ఉన్న కాగితపు సీలును చించండి. కాగితపు సీలులేని మరియు ఇదివరకే తెరిచి ఉన్న ప్రశ్నాపత్రమును మీరు అంగీకరించవద్దు.
   (ii) **కవరు పేజి పై ముద్రించిన సమాచారం ప్రకారం ఈ ప్రశ్న పత్రములోని పేజీల సంఖ్యను మరియు ప్రశ్నల సంఖ్యను సరిచూసుకోండి. పేజీల సంఖ్యకు సంబంధించి గానీ లేదా సూచించిన సంఖ్యలో ప్రశ్నలు లేకపోవుట లేదా నిజప్రతి కాకపోవుట లేదా ప్రశ్నలు క్రమపద్ధతిలో లేకపోవుట లేదా ఏవైనా తేడాలుండుట వంటి దోషపూరితమైన ప్రశ్న పత్రాన్ని వెంటనే మొదటి ఐదు నిమిషాల్లో పరీక్షా పర్యవేక్షకునికి తిరిగి ఇచ్చివేసి దానికి బదులుగా సరిగ్గా ఉన్న ప్రశ్నపత్రాన్ని తీసుకోండి. తదనంతరం ప్రశ్నపత్రము మార్చబడదు అదనపు సమయం ఇవ్వబడదు.**
   (iii) పై విధంగా సరిచూసుకొన్న తర్వాత ప్రశ్నాపత్రం సంఖ్యను OMR పత్రము పై అదేవిధంగా OMR పత్రము సంఖ్యను ఈ ప్రశ్నాపత్రము పై నిర్ధిష్ట స్థలంలో రాయవలెను.
4. ప్రతి ప్రశ్నకు నాలుగు ప్రత్యామ్నాయలు (A), (B), (C) మరియు (D) లుగా ఇవ్వబడ్డాయి. ప్రతి ప్రశ్నకు సరైన జవాబును ఎన్నుకొని OMR పత్రములో ప్రతి ప్రశ్నా సంఖ్యకు ఇవ్వబడిన నాలుగు వృత్తాల్లో సరైన జవాబు సూచించే వృత్తాన్ని బాల్ పాయింట్ పెన్‌తో కింద తెలిపిన విధంగా పూరించాలి
   ఉదాహరణ : (A) (B) ● (D)
   (C) సరైన ప్రతిస్పందన అయితే.
5. ప్రశ్నలకు జవాబును ఈ ప్రశ్నపత్రముతో ఇవ్వబడిన OMR పత్రము పైన ఇవ్వబడిన వృత్తాల్లోనే పూరించి గుర్తించాలి. అలాకాక సమాధాన పత్రం పై వేరొక చోట గుర్తించిన లేక సగ వృత్తం లేదా అసంపూర్ణ వృత్తాన్ని నింపిన మీ జవాబు మూల్యాంకనం చేయబడదు.
6. ప్రశ్న పత్రము లోపల ఇచ్చిన సూచనలను జాగ్రత్తగా చదవండి.
7. చిత్తుపనిని ప్రశ్నపత్రము చివర ఇచ్చిన ఖాళీ స్థలములో చేయాలి.
8. OMRపత్రము పై నిర్ణీత స్థలంలో సూచించవలసిన వివరాలు తప్పించి ఇతర స్థలంలో మీ గుర్తింపును తెలిపే విధంగా మీ పేరు రాయడం గానీ లేదా ఇతర చిహ్నాలను పెట్టడం గానీ చేసినట్లయితే మీ అనర్హతకు మీరే బాధ్యులవుతారు.
9. **పరీక్ష పూర్తయిన తర్వాత OMR పత్రాన్ని తప్పనిసరిగా పరీక్ష పర్యవేక్షకుడికి ఇవ్వాలి.** వాటిని పరీక్ష గది బయటకు తీసుకువెళ్ళకూడదు. పరీక్ష పూర్తయిన తరువాత అభ్యర్థులు ప్రశ్న పత్రాన్ని OMR పత్రం యొక్క కార్బన్ కాపీని తీసుకువెళ్ళవచ్చు
10. **నీలి/నల్ల రంగు బాల్ పాయింట్ పెన్ మాత్రమే ఉపయోగించాలి.**
11. **లాగరిథమ్ టేబుల్స్, క్యాలిక్యులేటర్‌లు, ఎలక్ట్రానిక్ పరికరాలు మొదలగునవి పరీక్ష గదిలో ఉపయోగించడం నిషేధం.**
12. **తప్పు సమాధానాలకు మార్కుల తగ్గింపు లేదు.**

DO NOT WRITE HERE

MATHEMATICAL SCIENCES
Paper – II

1. Which one of the following sets is countable ?

(A) [0, 1]

(B) $\mathbb{R}$

(C) The set of all polynomials with integer coefficients

(D) Cantor set

2. If $s_n = \left(1 + \frac{1}{n}\right) \cos n\pi$ then $\limsup\limits_{n \to \infty} s_n =$

(A) –1

(B) 0

(C) 1

(D) ∞

3. Which one of the following sets is a compact subspace of the set of all real numbers with the usual metric ?

(A) (– 2, 1]

(B) [4, ∞)

(C) [4, 5)

(D) [– 4, 0]

4. Let I = [0, 1] be the closed unit interval. Suppose f is a continuous mapping of I into I. Then

(A) $f(x) \neq x$ for all x in I

(B) $f(x) > x$ for all x in I

(C) $f(x) = x$ for at least one x in I

(D) $f(x) < x$ for all x in I

5. Which one of the following statements is not true ?

(A) Every convex function is continuous

(B) If f is a continuous mapping of a compact metric space X into a metric space Y then f is uniformly continuous

(C) Every continuous function is bounded

(D) If f is a continuous real function on a compact metric space X then f attains its bounds

6. Define $f : [1, 2] \to \mathbb{R}$ by $f(x) = x^2$, $x \in [1, 2]$.

Let $P = \left\{1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2\right\}$ be a partition of $[1, 2]$. Then $U(P, f) =$

(A) $\frac{50}{32}$

(B) $\frac{41}{16}$

(C) $\frac{85}{32}$

(D) $\frac{87}{32}$

7. $f(x) = \begin{cases} \frac{1}{\sqrt[3]{x}} & , \quad 0 < x \le 1 \\ 0 & , \quad x = 0 \end{cases}$

Then $\int_0^1 f(x)\,dx =$

(A) $\frac{1}{2}$

(B) 1

(C) $\frac{3}{2}$

(D) 2

8. Which one of the following is false ?

(A) If $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

then f has a discontinuity of the second kind at every point of x

(B) If $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

then f has discontinuity of the second kind at every point other than 0

(C) If $f(x) = \begin{cases} x^2 + 1 & \text{if } -2 < x < -1 \\ 3 + x & \text{if } -1 \le x < 0 \\ -4 + x^2 & \text{if } 0 \le x < 2 \end{cases}$

then f has a discontinuity of the second kind at $x = 0$

(D) If $f(x) = \begin{cases} \sin\frac{1}{x} & \text{if } x \ne 0 \\ 0 & \text{if } x = 0 \end{cases}$

then f has a discontinuity of the second kind at $x = 0$

9. Define $f : [0, 2] \to \mathbb{R}$ by $f(x) = 1 + e^x$, $x \in [0, 2]$.

Then the total variation of f on [0, 2] is

(A) 2

(B) $e^2 - 1$

(C) $e^2 + 1$

(D) $e - 1$

10. If $A : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation on $\mathbb{R}^n$ and if $\bar{x} \in \mathbb{R}^n$ then $A'(\bar{x}) =$

(A) A

(B) 0, the zero transformation

(C) 2 A

(D) $\frac{1}{2}$ A

11. Let $\mathbb{R}$ be the set of all real numbers and $d(x, y) = |x - y|$ for $x, y \in \mathbb{R}$. In the metric space $(\mathbb{R}, d)$ the derived set of the set $\mathbb{Z}$ of all integers is

(A) $\mathbb{Z}$

(B) $\mathbb{R}$

(C) $\mathbb{Q}$ the set of all rational numbers

(D) $\varnothing$ the empty set

12. Let (X, d) be a metric space and $\{K_n\}_{n=1}^{\infty}$ be a sequence of non-expanding non-empty compact subsets of X. Then $\bigcap_{n=1}^{\infty} K_n$ is

(A) equal to X

(B) equal to $\varnothing$, the empty set

(C) a non-empty compact set

(D) a compact set

13. Let w_1, w_2 be subspaces of a finite dimensional vector space V over a field F with dimensions 8, 6 respectively. If $w_1 \cap w_2 \neq \{0\}$ and m, n are the least and greatest possible values of dim $(w_1 + w_2)$, then $3m - n =$

(A) 7

(B) 9

(C) 11

(D) 13

14. Let $\phi : \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation defined by

$\phi((x_1, x_2, x_3, x_4)) = (x_1 + x_2 + x_3, x_2 + x_3, x_3 - x_4)$
for all $(x_1, x_2, x_3, x_4) \in \mathbb{R}^4$. Then the nullity of $\phi =$

(A) 0

(B) 1

(C) 2

(D) 3

15. Let A be a 4×4 matrix over the field $\mathbb{R}$ of all real numbers such that detA = 12. If ϕ is the Euler-totient function, then $\phi(\det(6A))$.

(A) 4836

(B) 8154

(C) 5184

(D) 3468

16. If the system of equations

$3x + 4y + 7z = 8$

$2x + 3y + \lambda z = 10$

$x + 2y + 4z = 6$

has no solution, then $4\lambda - 8 =$

(A) 14

(B) 12

(C) 8

(D) 0

17. If the inverse of the matrix $A = \begin{bmatrix} 3 & 4 & -1 \\ 4 & 1 & 2 \\ 5 & 0 & 8 \end{bmatrix}$ is $lA^2 + mA + nI$, then $l + m + n =$

(A) $\frac{13}{59}$

(B) $\frac{-23}{59}$

(C) $\frac{23}{59}$

(D) $\frac{-13}{59}$

18. If $T: \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation and B = {(1, 0, 0), (0, 1, 0), (0, 0, 1)} is the standard ordered basis of the vector space $\mathbb{R}^3$ over the field $\mathbb{R}$ such that $[T]_B = \begin{bmatrix} -1 & 0 & 2 \\ 7 & 3 & 1 \\ 4 & -6 & 5 \end{bmatrix}$, then T((6, 5, 7)) =

(A) (18, 29, 9)

(B) (8, 64, 29)

(C) (15, 34, 89)

(D) (12, 85, 72)

19. An orthonormal basis for the inner product space $\mathbb{R}^3(\mathbb{R})$ with respect to the standard inner product, among the following is

(A) $\left\{\left(\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}\right), \left(0, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right), \left(0, \frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)\right\}$

(B) $\left\{\left(\frac{3}{5}, \frac{4}{5}, 0\right), \left(\frac{-4}{5}, \frac{3}{5}, 1\right), \left(0, \frac{5}{\sqrt{34}}, \frac{-3}{\sqrt{34}}\right)\right\}$

(C) $\left\{\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right), \left(\frac{3}{5}, \frac{-4}{5}, \frac{-1}{5}\right), \left(\frac{4}{5}, \frac{2}{5}, \frac{-3}{5}\right)\right\}$

(D) $\left\{\left(\frac{-4}{5}, 0, \frac{3}{5}\right), (0, 1, 0), \left(\frac{3}{5}, 0, \frac{4}{5}\right)\right\}$

20. Let f be the symmetric bi-linear form defined on the vector space $\mathbb{R}^2(\mathbb{R})$ by $f((x_1, x_2), (y_1, y_2)) = (x_1 + x_2)(y_1 + y_2)$ for all $(x_1, x_2), (y_1, y_2)$ in $\mathbb{R}^2$ and q be the quadratic form associated with f. Then for $\alpha = (1, 5)$, $\beta = (2, 6)$, $q(2\alpha + 3\beta) =$

(A) 36 (B) 216

(C) 1296 (D) 108

21. For $|z| < 1$ we know $\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}$, using this result, the sum $\sum_{n=1}^{\infty} \frac{n^2}{3^n} =$

(A) $\frac{1}{2}$

(B) $\frac{3}{2}$

(C) 2

(D) 3

22. The Mobius transformation that maps i, 2, –2 onto i, 1, –1 respectively is

(A) $\frac{3+iz}{6z-4i}$ (B) $\frac{3z-2i}{iz-6}$

(C) $\frac{3z+2i}{iz+6}$ (D) $\frac{3z+2i}{6z+i}$

23. The function f(z) = cosz maps the line segment $\{(x, y) : -\pi < x \le \pi, y = 1\}$ onto

(A) a parabola

(B) an ellipse

(C) a circle

(D) a hyperbola

24. If $f(x, y) = \begin{cases} \frac{xy(x^2 - y^2)}{x^2 + y^2} & \text{for } x^2 + y^2 \neq 0 \\ 0 & \text{for } x^2 + y^2 = 0 \end{cases}$

then $\left(\frac{\partial^2 f}{\partial y \partial x}\right)_{(0, 0)} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)_{(0, 0)} =$

(A) 0 (B) 1

(C) – 2 (D) 2

25. If $a_n = \begin{cases} 2 & \text{for } n = 0 \\ 2^n & \text{for } n = 1, 2, 3, ... \end{cases}$

then the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n . Z^n$ is

(A) 1

(B) $\frac{1}{2}$

(C) 2

(D) $\frac{1}{3}$

26. The Laurent's series of $f(z) = \dfrac{1}{(z-1)(z-2)}$ which is valid for $1 < |z| < 2$ is

(A) $\displaystyle\sum_{n=-\infty}^{-1} \left(\frac{1}{2^{n+1}} - 1\right) z^n$

(B) $\displaystyle -\sum_{n=-\infty}^{-1} z^n - \sum_{n=0}^{\infty} \frac{1}{2^{n+1}} z^n$

(C) $\displaystyle\sum_{n=0}^{\infty} \left(1 - \frac{1}{2^{n+1}}\right) z^n$

(D) $\displaystyle\sum_{n=-\infty}^{\infty} \left(\frac{1}{2^{n+1}} - 1\right) z^n$

27. If $\mathcal{C}$ is given by the parametric equation $z(t) = t^2 + it$ for $0 \le t \le 1$ then $\displaystyle\int_{\mathcal{C}} z^2 dz =$

(A) $\dfrac{2}{3}(i-1)$

(B) $\dfrac{2}{3}(i+1)$

(C) $-\dfrac{2}{3}(i-1)$

(D) $-\dfrac{2}{3}(i+1)$

28. $\displaystyle\int_{|z|=2} \frac{z - 3\cos z}{\left(z - \frac{\pi}{2}\right)^2} dz =$

(A) $2\pi i$

(B) $4\pi i$

(C) $6\pi i$

(D) $8\pi i$

29. If $\mathcal{C}$ is a simple closed contour for which $z = 0$ lies inside and $z = 3$ lies outside it then $\displaystyle\int_{\mathcal{C}} \frac{dz}{z(z-3)} =$

(A) $\dfrac{2\pi i}{3}$

(B) $-\dfrac{2\pi i}{3}$

(C) $\dfrac{\pi i}{3}$

(D) $-\dfrac{\pi i}{3}$

30. For $0 < \lambda < 1$, $\displaystyle\int_0^{\infty} \frac{x^{\lambda-1}}{1+x} dx =$

(A) $\dfrac{\pi\lambda}{\sin(\pi\lambda)}$

(B) $\dfrac{\lambda}{\sin(\pi\lambda)}$

(C) $\dfrac{\pi}{\sin(\pi\lambda)}$

(D) $\dfrac{1}{\sin(\pi\lambda)}$

31. The number of bijections on the set {1, 2, 3, ..., 8} that are strictly increasing is

(A) 0

(B) 1

(C) $\frac{8!}{2}$

(D) 8!

32. The sum of all odd positive divisors of 1800 of

(A) 1331

(B) 304

(C) 3113

(D) 403

33. If ϕ is the Euler-totient function and if $\phi(12^4 \times 15^3)$ is $2^\alpha \times 3^\beta \times 5^\gamma$, then $\alpha + \beta + \gamma =$

(A) 16

(B) 17

(C) 18

(D) 19

34. The number of binary operations that can be defined on a set A with 6 elements is

(A) 6^6

(B) 6^2

(C) 6^{36}

(D) 36^6

35. In the symmetric group $(S_8, \circ)$, the product

$(1\ 4\ 6\ 8) \circ (3\ 4\ 2\ 1\ 6) \circ (4\ 8\ 6\ 3)^{-1} =$

(A) (1 8 2 4 6)

(B) (1 8 3 2 4)

(C) (1 8 4 3 2)

(D) (1 8 6 2 4)

36. If $\mathbb{Z}_{11}^* = \mathbb{Z}_{11} - \{\bar{0}\}$, then in the group $\left(\mathbb{Z}_{11}^*, X_{11}\right)$, the product $(\overline{11})X_{11}(\bar{7})^{-1}X_{11}(\bar{5}) =$ (where X_{11} is the multiplication modulo 11 on $\mathbb{Z}_{11}$) is

(A) $\bar{2}$

(B) $\bar{4}$

(C) $\bar{6}$

(D) $\bar{8}$

37. If the ideal n $\mathbb{Z}$ is a maximal ideal in the ring ($\mathbb{Z}$, +, .) of all integers , then a possible value of n is

(A) 57

(B) 67

(C) 77

(D) 87

38. The number of idempotent elements in the ring $(\mathbb{Z}_6, +_6, \times_6)$ of all residue classes of integers modulo 6 is

(A) 1

(B) 2

(C) 3

(D) 4

39. A polynomial over the field of rational numbers which is not irreducible among the following is

(A) $x^3 - 5x + 10$

(B) $5x^4 - 2x^3 + 6x^2 - 10x + 14$

(C) $x^4 - 3x^3 + 6x^2 - 9x + 5$

(D) $x^3 - 9x^2 + 3x + 12$

40. If K is the splitting field of the polynomial $x^4 - 3x^2 + 4 \in \mathbb{Q}[x]$, then the order of the Galois group $G(K/\mathbb{Q})$ is

(A) 1

(B) 2

(C) 4

(D) 8

41. Let $(\mathbb{R}, \tau)$ be the topological space of all real numbers with usual topology τ. Let $Y = [0, 1] \cup [2, 3)$ be equipped with subspace topology $\tau_Y = \{G \cap Y : G \in \tau\}$. Then the correct statement, among the following is

(A) $[0, 1] \in \tau_Y$

(B) $[2, 3) \notin \tau_Y$

(C) $[2, 3)$ is not closed in Y

(D) $[0, 1]$ is not closed in Y

42. Let $X = \{a, b, c\}$ and $\tau_1 = \{\varnothing, \{a\}, \{a, b\}, X\}$. In the topological space (X, τ), the interior of $\{b, c\}$ is

(A) $\varnothing$

(B) $\{a\}$

(C) $\{a, b\}$

(D) X

43. An example, among the following, a connected subset of the space $\mathbb{R}$ of real numbers with usual topology is

(A) $(1, 2) \cup (2, 3)$

(B) $[1, 2) \cup (2, 3]$

(C) $(1, 2] \cup [2, 3)$

(D) $(-1, 2) \cup (2, 4)$

44. The Wronskian of the functions $e^{at}\sin bt$ and $e^{at}\cos bt$ is

(A) e^{2at}

(B) be^{2at}

(C) $-be^{2at}$

(D) be^{at}

45. The solution of the initial value problem

$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 4y = 0,\ \ y(0) = 1,\ \ y'(0) = 1$ is

(A) $y(t) = e^{t}\left[\cos\sqrt{3}\,t + \frac{2}{\sqrt{3}}\sin\sqrt{3}\,t\right]$

(B) $y(t) = e^{-t}\left[\cos\sqrt{3}\,t + \frac{2}{\sqrt{3}}\sin\sqrt{3}\,t\right]$

(C) $y(t) = e^{-t}\left[\cos\sqrt{3}\,t - \frac{2}{\sqrt{3}}\sin\sqrt{3}\,t\right]$

(D) $y(t) = e^{-2t}\left[\cos\sqrt{3}\,t + \frac{2}{\sqrt{3}}\sin\sqrt{3}\,t\right]$

46. The solution of the system

$\frac{dx}{dt} = x,\ \frac{dy}{dt} = -x + 2y$ is

(A) $x = c_1e^t$
$y = c_1e^t + c_2e^{3t}$

(B) $x = c_1e^t$
$y = c_1e^t + c_2e^{2t}$

(C) $x = c_1e^t$
$y = c_1e^t + c_2e^t$

(D) $y = c_1e^t$
$y = c_1e^t + c_2e^{-t}$

47. The set of zeros of any non-trivial solution of the equation $y'' + (\sin^2 x + 1)y = 0$ has

(A) one element

(B) two elements

(C) finite number of elements

(D) infinitely many elements

48. The eigen values λ_n for the Sturm-Liouville problem $y'' + \lambda y = 0,\ y(0) = 0,\ \ y\left(\frac{\pi}{2}\right) = 0$ are

(A) $\frac{n^2}{4},\ n = 1, 2, ...$

(B) $\frac{n^2}{2},\ n = 1, 2, ...$

(C) $4n^2,\ n = 1, 2, ...$

(D) $8n^2,\ n = 1, 2, ...$

49. The integral surface of the equation $(2xy - 1)\,p + (z - 2x^2)q = 2(x - yz)$ which passes through the line $x_0(s) = 0$, $y_0(s) = s$ and $z_0(s) = 1$ is

(A) $x^2 + y^2 - y + z\,(1 - x) - 1 = 0$

(B) $x^2\,(1 - z^2) + (1 - 2xy)z - 1 = 0$

(C) $x^2 + y^2 + z = y + 2x$

(D) $x^2\,(1 - z^2) + z(1 - 2xy - x) - y = 0$

50. The general integral of

$2x(y + z^2)\, p + y(2y + z^2)\, q = z^3$ is

(A) $F\left(\frac{x}{yz}, \frac{z^2 - 2y}{yz}\right) = 0$

(B) $F\left(\frac{x}{yz}, (x-y)^2\ (x+y+z)\right) = 0$

(C) $F\left(\frac{x}{yz}, \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$

(D) $F\left(\frac{y}{z}, \frac{y^2}{x} - \frac{z}{x} - x^2\right) = 0$

51. The iteration formula $x_{n+1} = \frac{\alpha x_n + x_n^{-e} + 1}{\alpha + 1}$

(α any constant) is to compute a root of

(A) $x^3 - x^2 - \alpha = 0$

(B) $\alpha x^3 - x^2 - 1 = 0$

(C) $x^3 - x^2 - 1 = 0$

(D) $x^3 - \alpha x^2 + 1 = 0$

52. Consider the system of equations Ax = b where $A = \begin{bmatrix} 1 & -K \\ -K & 1 \end{bmatrix}$.

For which values of K, the Jacobi method converges ?

(A) $|K| > 1$

(B) $K = 1, -1$

(C) $|K| > 2$

(D) $|K| < 1$

53. The maximum value for step size h that can be used in the tabulation of f(x) = sinx in the interval [1, 3] so that linear interpolation will be correct to four decimal places after rounding is

(A) 0.2 (B) 0.25

(C) 0.02 (D) 0.5

54. Consider the function f (x) = ln x and its tabular values given below :

x :	2.0	2.2	2.6
f (x) :	0.69315	0.78846	0.95551

The derivative of f (x) at 2.0 is approximated by using linear interpolation. The value f′ (2.0) is

(A) 0.47655 (B) 0.49619

(C) 0.69182 (D) 0.73961

55. Consider the variational problem

$$V[y(x)] = \int_{x_0}^{x_1} (y + xy')\, dx,\ y(x_0) = y_0,\ y(x_1) = y_1$$

and the statements

P : Euler's equation reduces to the identity $1 \equiv 1$

Q : Since the integral does not depend on the path of integration, the variational problem is meaningless.

Which one of the following is true ?

(A) Both P and Q (B) Only P

(C) Only Q (D) Neither P nor Q

56. Consider the functional

$$V[z(x,y)] = \iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right] dxdy,$$

where D is the domain of z (x, y). In this problem, extremal satisfies

(A) Two dimensional wave equations

(B) Poisson equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = xy$

(C) Laplace equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

(D) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 1$

57. Consider the integral equation

$$F(x) = \lambda \int_a^x K(x, \xi)\, \phi(\xi)\, d\xi$$

and the statements

P : Above is Volterra's integral equation of first kind

Q : Above is Volterra's integral equation of second kind if $F(x) = \phi(x)$

Which one of the following is correct ?

(A) Only P

(B) Only Q

(C) Both P and Q

(D) Neither P nor Q

58. Resolvent Kernel of the integral equation

$$\phi(x) = x + \int_0^x (\xi - x)\, \phi(\xi)\, d\xi \text{ is}$$

(A) cos $(\xi - x)$

(B) sin $(\xi - x)$

(C) $e^{\xi - x}$

(D) $e^{x - \xi}$

59. A particle of mass m is moving on the inner surface of the cone $x^2 + y^2 = c^2z^2$ is

(A) not a mechanical system

(B) holonomic system

(C) non holonomic system

(D) neither holonomic nor non holonomic

60. Consider motion of a particle with mass m in polar coordinate system (r, θ). The force components are F_r and F_θ. The equation of motion is

(A) $F_r = m\ddot{r} - m\dot{\theta}^2 r, F_\theta = 2m\dot{r}\dot{\theta} + rm\ddot{\theta}$

(B) $F_r = m\ddot{r}, rF_\theta = 2m\dot{r}\dot{\theta} + rm\ddot{\theta}$

(C) $F_r = m\dot{\theta}^2 r,\ F_\theta = 2m\dot{r}\dot{\theta} + rm\ddot{\theta}$

(D) $F_r = m\ddot{r} - m\dot{\theta}^2 r,\ rF_\theta = 2r\dot{m}\dot{\theta}$

61. A certain treatment is used in 2 different centres A and B; patients in centre A were 25 and were an average 54 years old, patients treated in centre B were 62 and had mean age equal to 58 years. What is the overall mean among all patients who got the treatment ?

(A) 55.85

(B) 54.85

(C) 56.85

(D) 56.15

62. Pregnant women (within month 4) who are being followed up by nutrionist had weight (kg) equal to 64.3; 65.2; 70; 54.5; 54.5; 58.8; 81.5; 61; 62. What is the mean and the median ? Do this data suggest a strong skewness of the distribution of the weight ?

(A) Mean = 62.66, Median = 62.15, does not suggest that weight has a strongly skewed distribution

(B) Mean = 63.15, Median = 64.66, suggest a strongly skewed distribution

(C) Mean = 64.66, Median = 63.15, the data don't suggest that the variable weight has a strongly skewed distribution

(D) Mean = 64.66, Median = 63.15, the data don't suggest that the variable weight has no relevance of symmetricity

63. 3 candidates run for an election as a mayor in a city. According to a public opinion poll their chances to win are 0.25, 0.35 and 0.40. The chances that they build a bridge after they have been elected are 0.60, 0.90 and 0.80. What is the probability that the bridge will be built after the election ?

(A) 0.786

(B) 0.781

(C) 0.785

(D) 0.782

64. In an oral exam you have to solve exactly one problem. Which might be one of 3 types A, B & C. Which will come up with probabilities 30 %, 20 % and 50 % respectively. During your preparation you have solved 9 of 10 problems of type A, 2 of 10 problems of type B and 6 of 10 problems of type C. What is the probability that you will solve the problem of the exam ?

(A) 0.60

(B) 0.61

(C) 0.63

(D) 0.65

65. Let X be a random variable that follows a Poisson distribution with parameter λ = 7. What is the lower bound for $P(|X - \mu| \leq 4)$?

(A) 0.4375

(B) 0.4344

(C) 0.4295

(D) 0.4338

66. Let $X_1, X_2, \ldots, X_n$ be a sequence of i.i.d. random variables each with mean $E(X_i) = \mu$ and standard deviation σ, then SLLN is

(A) $P(\lim_{n\to\infty} \overline{X}_n = \mu) = 1$

(B) $P(\lim_{n\to\infty} \overline{X}_n = \sigma) = 1$

(C) $P(\lim_{n\to\infty} X_n = \overline{x}) = 1$

(D) $P\left(\dfrac{X_n}{n} = \overline{x}\right) = 1$

67. Suppose the age of a student graduated from a state is normally distributed. If the mean age is 23.1 years and s.d. is 3.1 years. What is the probability that 6 randomly selected students had a mean age at graduation that was greater than 27 ? (Given the table value = 0.9990)

(A) 0.0001

(B) 0.0011

(C) 0.0005

(D) 0.0100

68. The Cauchy random variable is defined to be the ratio of 2 independent standard Normal variables X & Y with probability density function

(A) $f_v(u) = \dfrac{1}{\pi(\nu^2+1)}\ ; -\infty < \nu < \infty$

(B) $f_v(u) = \dfrac{\pi}{(\nu^2+1)}\ ; -\infty < \nu < \infty$

(C) $f_v(u) = \dfrac{XY}{\pi(\nu^2+1)}\ ; -\infty < \nu < \infty$

(D) $f_v(u) = \dfrac{\pi(\nu^2+1)}{XY}\ ; -\infty < \nu < \infty$

69. The sample paths of a Markov chain are completely characterised by the

(A) Normal Transition Probabilities

(B) One step Transition Probabilities

(C) Two step Transition Probabilities

(D) n-step Transition Probabilities

70. An irreducible Markov Process is a Markov Process for which the embedded Markov chain is

(A) Transition Probability Matrix

(B) Birth and Death Process

(C) Irreducible

(D) Poisson Process

71. Let m and n be positive Integers, then $\sum_{r=0}^{K}\binom{m}{r}\binom{n}{K-r} = ?$

(A) $\binom{m+n}{K}$

(B) $\binom{m}{K}$

(C) $\binom{m-n}{K}$

(D) $\binom{m-1}{K-1}$

72. Let X ~ Exp (λ) using Markov inequality. What is the upper bound for P(X > a) ?

(A) $\frac{1}{\lambda a}$

(B) $\frac{1}{\lambda}$

(C) $\frac{1}{\lambda^2}$

(D) $\frac{1}{(\lambda a)^2}$

73. If X ~ Exp (λ) using Chebyshev's Inequality, what is the upper bound for $P(|X - EX| \geq b)$?

(A) $\frac{1}{\lambda b}$

(B) $\frac{1}{\lambda^2 b^2}$

(C) $\lambda^2 b$

(D) $\frac{1}{\lambda b^2}$

74. Let X_1, X_2, X_3 be a random sample from a distribution of the continuous type having probability density function

$$\begin{aligned} f(x) &= 2x; \quad 0 < x < 1 \\ &= 0; \quad \text{O.W.} \end{aligned}$$

What is the probability that the smallest of X_1, X_2, X_3 exceeds the Median of the distribution ?

(A) $1/8$

(B) $1/7$

(C) $1/6$

(D) $2/8$

75. Let X_n is an i.i.d. exp (λ), then derive the p.d.f. of $X_{(n)}$

(A) $n\{1 - e^{-\lambda x}\}^{n-1} \lambda e^{-\lambda x}; x > 0$

(B) $n\{1 - e^{-\lambda x})^{n-1}; x > 0$

(C) $e^{-\lambda x}, \lambda e^{-\lambda x}; x > 0$

(D) $e^{-\lambda x}/\lambda e^{-\lambda^2}; x > 0$

76. If $H_0 : x \sim$ Poisson (λ_0) ;

$H_1 : x \sim$ Poisson (λ_1) ; $\lambda_1 > \lambda_0$

$\wedge(x) = \bar{e}^{(\lambda_1 - \lambda_0)} (\lambda_1 / \lambda_0)^x$ is a

(A) Simple hypothesis for any $\lambda_1 > \lambda_0$, λ_0 pair

(B) Non decreasing function in x for any $\lambda_1 > \lambda_0$, λ_0 pair

(C) Does not exist for any $\lambda_1 > \lambda_0$, λ_0 pair

(D) Composite hypothesis for any $\lambda_1 > \lambda_0$, λ_0 pair

77. Suppose $X_1, X_2, ..., X_n$ is a random sample from a normal distribution with mean 'μ' and unknown variance 'σ^2'. The null hypothesis $\mu = 12$ is a

(A) Simple hypothesis

(B) Both simple and composite hypothesis

(C) Composite hypothesis

(D) Not a hypothesis

78. Let $X_1, X_2, ..., X_n$ be a Bernouli random variables with parameter 'p'. What is the method of moments estimation of p ?

(A) $\sum_{i=1}^{n} X_i$

(B) $\frac{1}{n}\sum_{i=1}^{n} X_i^2$

(C) $\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$

(D) $\frac{1}{n}\sum_{i=1}^{n} X_i$

79. If (P, Q) has a Bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1, \sigma_2$ and p then the variance of the conditional distribution of (Y/X = x) is

(A) $\sigma_1^2(1 - p^2)$

(B) $\sigma_2^2\ p^2$

(C) $\sigma_2^2\ (1 - p^2)$

(D) $\sigma_1^2\ p^2$

80. Given the J.P.d.f. of X and Y as

$$f(x,y) = 4xy; \quad 0 \le x \le 1; 0 \le y \le 1$$
$$= 0 \quad ; \quad \text{Elsewhere,}$$

then what is the value of

$$p\left\{0 < x < \tfrac{1}{2};\ \tfrac{1}{2} \le y \le 1\right\}$$

(A) $3/16$

(B) $5/16$

(C) $3/8$

(D) $1/4$

81. If T_1 and T_2 are independent and unbiased estimators of parameter 'θ' with $V(T_1) = \sigma_1^2$ and $V(T_2) = 2\sigma_1^2$, then unbiased estimate of θ given by $T = \lambda_1 T_1 + (1 - \lambda)T_2$, minimum variance if λ equals

(A) $1/2$

(B) $1/3$

(C) $1/4$

(D) $2/3$

82. You have conducted a study comparing Army, Navy and RAF cadets on a measure of leadership skills. These are unequal group sizes and the data is skewed so you need to use a non parametric test, then which test you choose ?

(A) Friedman test

(B) Mann Whitney test

(C) Kruskal Wallis test

(D) Wilcoxon Rank Sum test

83. One or two tail tests will determine

(A) If the two extreme values (minimum or maximum) of the sample need to be rejected

(B) If the hypothesis has one or possible two conclusions

(C) If the region of rejection is located in one or two tails of the distribution

(D) We accept the test

84. If the ties occur in the Kruskal Wallis test with usual notations, the correction 'C' for ties is

(A) $\Sigma T/K(n^2 - 1)$

(B) $\Sigma T/(n^2 - 1)$

(C) $\Sigma T/n(n - 1)$

(D) $\Sigma T/(n^2 - 1)\ n$

85. What do you report in a maximum likelihood ratio to say whether your model was significant or not ?

(A) ANOVA

(B) Correlation

(C) R-squared

(D) Beta

86. The basic assumption behind regression analysis is

(A) To estimate a line that goes through most values in the observed data

(B) To minimise the sum of the squared residuals

(C) Estimate a line that maximises the difference between the $\sum_{i=1}^{n}(Y_i - \hat{Y}_i)$

(D) All the above

87. What assumptions does ANCOVA have that ANOVA does not have ?

(A) Homogeneity of variance

(B) Homoscedasticity

(C) Homogeneity of sample size

(D) Homogeneity of regression slopes

88. A logistic regression model was used to assess the association between C.V.D. and obesity. p is defined to be the probability that the people have CVD, obesity was coded as o = non obese, log (p/1 – p) = – 2 + 0.7 (obesity). What is the log odds ratio for CVD in persons who are obese as compared to not obese ?

(A) 0.7

(B) –2

(C) 2.7

(D) exp (0.7)

89. In simple logistic regression the traditional goodness of fit measure, – 2 (log likelihood of current model – log likelihood of previous model) is

(A) A statistic that does not follow a χ^2 p.d.f.

(B) Indicates the spread of answers to a question

(C) An index of how closely the analysis reaches statistical significance

(D) How close the predicted findings are to actual finding

90. Match the following :

1. Rotated factor loading	a. This is the number of factors plotted against variance
2. The scree plot	b. Carried out simply reduce a larger data set to a smaller one
3. Principles component analysis	c. You look at this to name a factor
4. Correlation matrix	d. The first step taken to perform a factor analysis

(A) 1 – c, 2 – a, 3 – b, 4 – d

(B) 1 – a, 2 – d, 3 – b, 4 – c

(C) 1 – c, 2 – d, 3 – b, 4 – a

(D) 1 – b, 2 – a, 3 – d, 4 – c

91. Which of the following is finally produced by hierarchical clustering ?

(A) Final estimate of cluster centroids

(B) Tree showing how close things are to each other

(C) Assignment of each point to cluster

(D) All the mentioned above

92. In sampling without replacement, the standard error of the sample mean vanishes if

(A) $n = N$

(B) $n > N$

(C) $n < N$

(D) $n \neq N$

93. Let 'μ' be the population mean and 'σ^2' be the population variance and a sample of size 'n' is drawn by simple random sampling with replacement from a population of size N. Let $Y = \sum_{i=1}^{n} \lambda_i x_i$ be a best linear unbiased estimator, then the variance of Y is equal to

(A) $\sigma^2\left(1+\frac{1}{n-1}\right)\sum_{i=1}^{n}\lambda_i^2+\frac{\sigma^2}{N-1}\sum_{i=1}^{n}\lambda_i^2$

(B) $\frac{\sigma^2}{N-1}\sum_{i=1}^{n}\lambda_i^2+\sigma^2\left(1+\frac{1}{N-1}\right)\left(\sum_{i=1}^{n}\lambda_i^2\right)$

(C) $\frac{\sigma^2}{n-1}\sum_{i=1}^{n}\lambda_i^2-\sigma^2\left(1+\frac{1}{N-1}\right)\sum_{i=1}^{n}\lambda_i^2$

(D) $\sigma^2\left(1+\frac{1}{N-1}\right)\sum_{i=1}^{n}\lambda_i^2-\frac{\sigma^2}{(N-1)}\left(\sum_{i=1}^{n}\lambda_i^2\right)$

94. Let N be number of units in a population. After the selection of one unit from a population, every K^{th} ($K < n$) unit is selected to obtain a sample of size n. Let P be the inter class correlation between the units of the same systematic sample. If P = 1, then the relative precision of systematic sample with simple random sampling is

(A) a function of N only

(B) a function of N and K only

(C) a function of N and n only

(D) a function of N, n and K

95. For a BIBD with parameters v = b = 7, r = k = 4, λ = 2, the number of treatments common between any two blocks is

(A) 3

(B) 4

(C) 1

(D) 2

96. In a 2^3 factorial experiment the principal blocks of replicates 1 and 2 respectively consist of {(1), A, BC, ABC} and {ABC, AC, B, (1)}. What are the confounded interaction effects in the 2 replicates respectively ?

(A) BC and AC

(B) ABC and AB

(C) BC and AB

(D) AB and BC

97. Let R(t) and h(t) respectively denote the reliability and hazard rate of a unit at time t, then which of the following relation is correct ?

(A) $\log R(t) = \int_0^t r(x)\,dx$

(B) $\log R(t) = -\int_0^t r(x)\,dx$

(C) $\log R(t) = 1-\int_0^t r(x)\,dx$

(D) $\log R(t) = \exp\left\{-\int_0^t r(x)\,dx\right\}$

98. Consider the L.P.P.

Max Q = $x + 3y + 6z - w$

Subject to $5x + y + 6z + 7w \leq 20$;

$6x + 2y + 2z + 9w \leq 40$

$x, y, z, w \geq 0$

(A) 55

(B) 58

(C) 60

(D) 62

99. If X(t) is the number of customers in an M/M/1 queuing system with arrival rate $\lambda > 0$ and service rate $\mu > 0$, then what is the process $\{X(t), t > 0\}$ is

(A) Birth process with $(\lambda - \mu)$

(B) Poisson process with rate $(\lambda - \mu)$

(C) Markov process

(D) Birth and death process

100. In a linear programming problem with 4 constraints and 6 variables which of the following is equal to the maximum number of basic solutions ?

(A) 4

(B) 10

(C) 24

(D) 15

Space for Rough Work

Space for Rough Work

Space for Rough Work